IB DP Unit Planner

Probability HL

TEACHER
Poorvi Doshi

SUBJECT
Mathematics

SHARED GRADES
Grade 12

START DATE
Week 3, April

DURATION
6 Weeks

COURSE PART
5.2,5.3,5.4,5.5,5.6,5.7

UNIT DESCRIPTION
Definition of Probability, the laws of probability (5.2–5.4) - Addition and Multiplication Rule, and random variables and their probability distributions (5.3, 5.4) - Binomial, Poisson-(5.5,5.6) and Normal Distribution(5.7)

INQUIRY & PURPOSE

Essential Understandings

- Difference between theoretical and experimental probability.
- Distinguish between an event having zero probability and an event being impossible.
- How can students make predictions based on data?
- Is estimation more appropriate than finding an exact answer?
- What determines whether an event is dependent or independent?
- What are the different Probability Distributions?

Inquiry Questions

Skills-based	What is the importance and value of Statistical thinking in problem-solving in a variety of discipline(Economics, Chemistry, Physics, biology, and Psychology)?
Content-based	What is the difference between theoretical and experimental probability?
Content-based	How do students know whether an event is independent or dependent?
Concept-based	How do students know which is the best distribution to model the data ?

Transfer goals

The student should be able to apply
laws and theorems of probability theory
conditional probability and conditional probability (law of total probability, Bayes' theorem).
Model, Interpret and analyze the data using different distributions.

Missed concepts/misunderstandings

- Outcomes of Random Experiments i.e n(S)= total number of equally likely outcomes.
- Students often misunderstand between independent events and disjoints sets

Probability HL

RESOURCES

Photograph of coin and dice activity

Added by **Poorvi Doshi** on June 30, 2019

Photograph of coin and dice activity

IMG20190629125043.jpg

Added by **Poorvi Doshi** on June 30, 2019

Photograph of coin and dice activity

IMG20190629125048.jpg

Added by **Poorvi Doshi** on June 30, 2019

Photograph of coin and dice activity

Photograph of coin and dice activity

Added by **Poorvi Doshi** on June 30, 2019

Unit-test Probability

Added by **Poorvi Doshi** on June 30, 2019

Probability Test

Added by **Poorvi Doshi** on June 30, 2019

TOK - Article related to Intelligence quotient (Kognity)

https://scholar.flatworldknowledge.com/books/127/preview

Added by **Poorvi Doshi** on June 28, 2019

Continuous Distribution - Normal Distribution

https://docs.google.com/document/d/1lrxQ8Q382i-mCJwFhm42TCV6XsZjFdQXVIxEXiATO0I/edit

Added by **Poorvi Doshi** on June 25, 2019

Lesson Plan - Normal Distribution

Discrete Probability Distribution

https://docs.google.com/document/d/1bS6pPMKcnMahHAmGh6psdtl37Ybwh0XkuGahnx422So/edit
Added by **Poorvi Doshi** on June 25, 2019
Lesson Plan - Discrete Probability Distribution

Basic Probability

https://docs.google.com/document/d/1YxgoBpVQvc96gcuJ4ApPNDPJ71hMZ8DwhUukDzC0bOo/edit
Added by **Poorvi Doshi** on June 25, 2019
Lesson Plan - Basic Probability

TOK In Probability

https://scholar.flatworldknowledge.com/books/127/preview
Added by **Poorvi Doshi** on June 25, 2019
Intelligence quotient (IQ) distributions offer a TOK-related real-life situation that assumes intelligence can be measured. The following link explains how IQ testing connects to the normal distribution, and what assumptions are made. It also explores how intelligence testing can be biased, and how speaking of intelligence as a single entity ignores theories regarding multiple intelligences. (Kognity)

TOK In Probability

https://drive.google.com/drive/folders/13laNTgcSssEasWzqOMux8uet8zjAkjqe
Added by **Poorvi Doshi** on June 25, 2019
Classroom video on TOK, MATHEMATICS and PSYCHOLOGY

Photograph Yahtzee Game

Added by **Poorvi Doshi** on June 24, 2019

Photograph Yahtzee Game

Added by **Poorvi Doshi** on June 24, 2019

Introduction To normal Distriubtion

https://www.khanacademy.org/math/statistics-probability/modeling-distributions-of-data/more-on-normal-distributions/v/introduction-to-the-normal-distribution
Added by **Poorvi Doshi** on June 24, 2019

Statistics 101/A Tour of the Normal Distribution

https://www.youtube.com/watch?v=772_n15Ke9Q
Added by **Poorvi Doshi** on June 24, 2019
This video gives an in-depth view of the normal distribution including its characteristics and associated cumulative probabilities.

HL_5.11_POISSON_DISTRIBUTION.pdf

Added by **Poorvi Doshi** on June 3, 2019

HL_5.4-5.7_PROBABILITY.pdf

Added by **Poorvi Doshi** on June 3, 2019

HL_5.9_BINOMIAL_DISTRIBUTION.pdf

Added by **Poorvi Doshi** on June 3, 2019

HL_5.8_DISTRIBUTIONS_IN_GENERAL__DISCRETE_AND_CONTINUOUS_.pdf

Added by **Poorvi Doshi** on June 3, 2019

CURRICULUM

Aims & Objectives

AIMS

Enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
Develop an understanding of the principles and nature of mathematics
Develop logical, critical and creative thinking, and patience and persistence in problem-solving
Appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics
Appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course

OBJECTIVES

Knowledge and understanding: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts
Problem-solving: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems
Technology: use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems

Syllabus Content

Mathematics HL Core

- 5 - Statistics and probability

- 5.2 Concepts of trial, outcome, equally likely outcomes, sample space (U) and event. The probability of an event A as $P(A) = \frac{n(A)}{n(U)}$. The complementary events A and A′ (not A). Use of Venn diagrams, tree diagrams, counting principles and tables of outcomes to solve problems.
- 5.3 Combined events; the formula for $P(A \cup B)$. Mutually exclusive events.
- 5.4 Conditional probability; the definition $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. Independent events; the definition $P(A \mid B) = P(A) = P(A \mid B')$. Use of Bayes' theorem for a maximum of three events.
- 5.5 Concept of discrete and continuous random variables and their probability distributions. Definition and use of probability density functions. Expected value (mean), mode, median, variance and standard deviation. Applications.
- 5.6 Binomial distribution, its mean and variance. Poisson distribution, its mean and variance.
- 5.7 Normal distribution. Properties of the normal distribution. Standardization of normal variables.

Content, Skills & Concepts

CONTENT

- Define the sample space.
- Recognize that the sum of the probabilities of the distinct outcomes within a sample space is one.
- Define the complement of an event.
- Describe the formula for finding the probability of the complement of an event.
- Recognize the relationship between the sample space of an experiment and the complement of an event.
- Define mutually exclusive events.
- Define dependent events and conditional probability.
- Examine experiments without replacement.
- Explain the relationship between conditional probability and dependent events.
- Interpret the derivation of the conditional probability formula using set theory and Venn diagrams.
- Sums on Conditional probability and independent events.
- Understand the concepts of a random variable and a probability distribution.
- Be able to distinguish between discrete and continuous random variables.
- Be able to compute and interpret the expected value, variance, and standard deviation for a discrete random variable.
- Be able to compute probabilities using a binomial probability distribution.
- Be able to compute probabilities using a Poisson probability distribution.
- Understand the difference between how probabilities are computed for discrete and continuous random variables.
- Know how to compute probability values for a continuous uniform probability distribution and be able to compute the expected value and variance for such a distribution.
- Be able to compute probabilities using a normal probability distribution. Understand the role of the standard normal distribution in this process.

SKILLS

- Collect and analyze data.
- compute the probability of a mutually exclusive event.
- compute the probability of a not mutually exclusive event through example.
- connect set theory and Venn diagrams with events that are mutually and non-mutually exclusive, addition rule, the complement of a set.

CONCEPTS

Students will understand

- the conversion between decimal, percentage, and fraction representations of a rational number including graphically.
- What does a probability of zero represent?
- What does a probability of one represent?

- What does a probability between zero and one represent?
- What method is best suited for the data set?

ASSESSMENT

Formative assessment

Guided Practice - Students will be guided and monitored throughout the lesson with questions and problems. Students will take notes for the topic. The teacher will model correct answers on the board or ask student volunteers to model the correct answers on the board. The teacher will use a combination of these two methods.
Independent practice(Homework) - This will build upon the skills taught to move students to master the concept. It is to solve practice problems like the problems on the final summative Assessment.

Summative assessment

Unit test on Probability - Teacher will ask students higher order thinking questions regarding the topics taught under the title of Probability. The teacher will grade students' work based on the summative assessment answer key.

Peer and self assessment

Students will discuss assignments given at home with peers and teacher.

Assessment criteria

HL Criteria

Internal Assessment

Mathematical exploration

B: Mathematical presentation

E: Use of mathematics

External Assessment

Paper 1 (No calculator allowed)

A: Short-response questions based on the core syllabus

B: Extended-response questions based on the core syllabus

Paper 2 (Graphic display calculator required)

A: Short-response questions based on the core syllabus

B: Extended-response questions based on the core syllabus

LEARNING EXPERIENCES

Prior learning experiences

Activity for the introduction. Students have prior knowledge of some ideas and notations of the topic
of probability from their IGCSE curriculum. The teacher will revisit the topic to ensure that all students recall back the basic definition of probability. Activity for introduction will ensure the required understanding of the basic concept and connecting the information gathered during the experimentation to brush upon their old ideas.

Pedagogical approaches

- Inquiry-based learning
- conceptual understanding
- Chalk and talk

Feedback

The teacher will use 'Quiz format' and 'questioning for understanding' to give specific and immediate feedback to students. Questionnaire regarding whether the students have understood the concept.

Student expectations

To an extent rely on the unit test to measure their understanding. Homework and classwork assignments are the major evidence of the students that reflects students understanding. Short quizzes at the beginning of the next class to ensure revision and understanding before proceeding with the next topic.

Support materials

Examples
Sample Exam Questions
Mark Schemes

Learning Process

Lecture
Small group/pair work

Differentiation

Scaffold learning
Extend learning

Activity for Introduction -

At the beginning of the lesson, students will conduct an experiment. I will ask students to flip a coin and two dice 500 times among the eight groups. I divided sixteen students into eight groups of two each. The five groups had to record the outcome (head or tail) for 100 times. Rest of the groups recorded the outcomes on two dice. Complied the data and analyzed the difference between theoretical and experimental probability. This will allow me to assess them on the prior knowledge necessary for this lesson.

Game - Students played a game after understanding the concept of conditional probability. The game was an initiation to the concept of the binomial distribution. Students played Yahtzee game.

Yahtzee Game - 16 players - 5 teams with three in each and two in a single team. Top three Yahtzee cards a replaced on the gameboard after shuffling the card deck. Chips are placed on the circles on the game board. Each team plays with five dice. The deck of Yahtzee cards has a variety of three sets - Number, Combo cards and Yahtzee cards. Each team rolls five dice and based on the outcomes either take a card from the gameboard or steals a card from the opponent’s Home space. Along with the card, the players will take a chip. The maximum number of points wins the game. If the outcomes do not match card on the board game it better luck next time.

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CONNECTIONS

Approaches to Learning

- **Thinking**
- **Social**
- **Self management**
- **Research**

Thinking skills - Students will plan a reasoned argument to support his/her conclusion or evaluation.

Self Management - The teacher will ask students to allocate among themselves, different roles in a classroom discussion or in an activity.

Social - Students will work in groups for most of the activities. They will assess their peer groups for performance and process.

Learner Profile

Inquirers

The topic probability develops their natural curiosity.

Knowledgeable

The student explore concepts and ideas that the Psychology teacher discussed in the class.

Thinkers

The students apply thinking skills critically and creatively to recognize and approach a problem(Yahtzee game), and while playing the game the students were making some reasoned choices whether to snatch the cards from a teammate or to take a card from the board game.

International Mindedness

De Moivre's derivation of the normal distribution and Quetelet's use of it to describe?

How is randomness and religion today in Burkina Faso, western Africa?

Information Communication Technology

Information literacy will play a major role when students choose a topic for an internal assessment/ extended essay that involves the use of statistics. Using the right tools in spreadsheets will aid in collecting/analyzing the information(data collection).

Language and learning

Activating background knowledge
Acquisition of new learning through practice

TOK Connections

Personal and shared knowledge
Ways of knowing
The knowledge framework
To what extent can we trust mathematical models such as the normal distribution?

REFLECTIONS & EVALUATION

2019

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Prior to studying the unit

I designed an activity to test prior knowledge of probability. To begin the activity, in one group each student tosses one coin. In pairs, both students toss the coin 50 times each and record the results. The other group rolled a die 100 times. The activity is designed to teach and reinforce the concept of basic probability.

During the unit

- The student began to realize that experimental probability is close to the theoretical probability.
- The derivation of the mean of the binomial distribution, which reduces to 'np' was very intriguing for students.
- I had planned to show a video on the "Monty Hall" problem and wanted to discuss whether the 'Monty Hall problem is correct and what aspect of probability is taking into consideration while discussing the problem?". For unavoidable circumstances, the vacation got extended and could not allocate the same number of class to Probability as it was planned.

Notes/changes/suggestions:

Would like to plan a lesson which would integrate other subjects like Biology that involves statistical analysis. Use of probability methods in medical studies to assess risk factors for certain diseases. The Algebra unit has an inquiry-based activity for Superannuation. Maybe could extend it further to calculate the expected gain to insurance companies.

What worked well

The students began to use probability in daily life to decide when they don't know for sure what the outcome will be. Most of the time, they don't perform actual probability problems but use subjective probability to make judgment calls and determine the best course of action. This was evident when they were discussing which team would win the Cricket world cup 2019.

What didn’t work well

It was vital for me to understand students as learners. It is essential while designing the lesson plan to take into consideration the different needs of the students within the class. Few students understood the material content, but it did not mean that as a teacher, I was meeting their needs as learners.
International Mindedness - There were two questions identified in the planning for international mindedness. The students' could not touch upon as per the plan because of the time crunch.

Transfer reflection

Rolling the Dice

We did an activity with two dice in the class, as an introductory activity to probability. We did 150 trials wherein we rolled two dice and wrote the outcomes. Then we calculated the probability of both dice showing the same number. This activity was a fun introduction to probability because I learned that the number of trials one does, the closer they come to the theoretical probability. We compared the actual probability calculated through our trials and compared it to the theoretical probability and learned that if we increased the number of trials, we would come closer to the actual value. The activity was an effective way to introduce the concept of probability.
Aarya Kamdar

Yahtzee game
In our maths class to further learn probability, we played a game known as Yahtzee. We first were divided into groups of 3 and learned the rules of the game. After playing the game for a while, we found out that the game has multiple application of probability. So we in teams strategized to win over the others by trying to use probability. At first, we only looked to earn cards which were very hard to get as they were scoring high cards. But we soon realized that if another team was to get such dice roll, we wouldn't be able to get that card again, as the probability of it was very low. Due to this we tried to win moderate cards and tried to team up with one another team, as it would make both the teams stronger while the others were weaker. This made the probability of winning more. Afterward, we always tried to steal cards rather than bring new cards from the board, as it would also give other teams more chance of stealing more of our cards. Finally, after many rounds of the game, our team won by teaming up with another and stealing cards from the other 2 teams. We barely won against our allies by just based on our points. This showed many applications of probability, like conditional probability and much more. Moreover, it was a very fun experience to learn by playing such a competitive game.
Malay Sagparia