DP Unit Planner - Algebra

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INQUIRY: Essential Questions

Transfer goals Transfer goals are the major goals that ask students to “transfer” or apply, their knowledge, skills, and concepts at the end of the unit under new/different circumstances, and on their own without scaffolding from the teacher.
Through Algebra student should be able to appreciate the generalization of numbers. Algebra should help students understand the instructions and relations for the unknowns. Why is Algebra important for study of Sequence and Series? Perceive the power of Mathematical notation. How are rational exponents linked to scientific notation? Why do we need complex numbers?

Topic	Details
Sequence and Series	• Arithmetic sequences & series; sum of finite arithmetic sequences • geometric sequences & series; sum of finite & infinite geometric sequences • sigma notation • problems involving compound interest & other examples of exponential growth & decay
Exponents and Logarithms	• Exponential functions; laws of exponents; graphs of exponential functions; exponential growth & decay; the number e • Logarithmic functions; properties of logarithms; change of base; solving exponential and logarithmic equations
Counting principles, including permutations and combinations	• Factorial notation • Permutations & combinations • expansion of binomial expressions; binomial theorem; binomial coefficients; • Pascal's triangle • Proof by mathematical induction
Binomial Theorem	
Proof by mathematical induction.	
Complex Numbers	• Complex numbers: the number i = $\sqrt{-1}$; the • terms real part, imaginary part, conjugate, modulus and its simple properties. • Cartesian form z = a + ib • Sums, products and quotients of complex numbers.

ACTION: teaching and learning through inquiry

Content/skills/concepts—essential understandings	Learning Process
<u>Students will know the following content:</u> • Students will be able to solve a variety of exponential and logarithmic equations. • Students will be able to set up exponential growth and decay models and to solve the associated word problems. • Arithmetic sequences and series; sum of finite Arithmetic series; geometric sequences and series; sum of finite and infinite geometric series. • What happens when binomial is multiplied by itself n numbers of times? • Understanding Pascal's triangle • Students develop an understanding of combinatorial reasoning using concepts of permutation and combination • Students will be familiarised with Complex Numbers: algebraic and geometric interpretation:	Learning experiences and strategies/planning for self-supporting learning: ➢ Inquiry Based Learning ➢ Investigation ➢ Lecture ➢ Small group/pair work Details: **<u>Inquiry based Learning (PBL)</u>** **Question:** 'How will superannuation help in understanding pension plans a person in TGES?'(Project based learning)' Through this project students will extent knowledge of geometric progression to superannuation through a meaningful project. They will analyse pension plans and prepare an end product which could be flyer, PowerPoint presentation, brochure.

Operations on complex numbers. Conjugate and Modulus and its properties. **Students will develop the following skills**: • Students will develop abstract reasoning skill/sense. • Students should be able to conceptualize the problem. • Students should be able to strategize solutions for a given problem. • Students will be able to graph exponential and logarithmic functions. • Construct and compare linear, quadratic, and exponential models and solve problems. • Simplify expressions containing rational exponents. • Evaluate expressions with irrational exponents. • To extend the knowledge of series in understanding recurrence sequences (e.g Fibonacci Sequences) • Students will develop skills of proving mathematical propositions specially the combinatorial identities using induction. **Students will grasp the following concepts:** • Finding from the 'known to unknown' and 'unknown to known' • Understanding Notations (Using of symbols) • Sigma notation for summation • Infinity • Understanding Patterns • Relation between the variables • Generalization • Introduction to concept of Infinity • Concept of inverse function • Induction is a defining difference between discrete and continuous mathematics	This project will help students to translate their mathematical competency to a real time application. As part of the project students will be introduced to literary meaning of superannuation and currently the pension plans existing in the market. **Investigation** Introduction to Pascal's triangle and making connections with Binomial theorem through investigation. **Lecture Method** • Counting Principles – Permutation and Combination • Mathematical Induction

Component	Details
Language and Learning	
Scaffolding for new learning	• A senior faculty of humanities, Ms Parul Modha will be inviting to exchange her views about annuity from literary, history and mythological point of view. The students should experience elegance of literature in defining the semantic of the word annuity in a larger context.
Acquisition of new learning through practice	• Invite Mr.Nilesh Ladani a financial advisor to brief students about the importance of saving and investing early in life. • Ms. Khushboo Naidu – Economics teacher will talk about terminal annuities. Public borrowing in modern time is necessary to meet various essential requirements. There are multiple methods of debt redemption, one being terminal annuities. Under this method, the government pays its debt in equal, which include the interest as well as the principal amount of debt. The government is not required to repay its entire debt at a time, but the burden of debt is reduced every year. Thus, it is the method of repayment of loans in installments. The weight of the debt goes on diminishing annually, and by the time of maturity, it is already fully paid off. • *Reference used: ISC Economics, Frank publications* • *Authors: D.K. Sethi and U Andrews.*
CAS connections	
Service	Through this project students will extend knowledge of geometric progression to annuities through a real-time project. They will analyse the pension plans for retirement for a person of any age group in TGES. This project will help students to translate their mathematical competency to a real-time application. They will model as financial advisors to our staff by condensing the information about a particular plan in a presentation that could be using the flyer, PowerPoint, spreadsheet, etc.
Creativity	**Pop-out cards Activity***(@https://fractalfoundation.org/resources/fractivities/fractal-cutout/)* Students will make a 3-dimensional fractal pop out card by using the process of cutting and folding. The student will attempt to write a recursive formula for the pattern being used. These cards will be used as "thank you cards" or invitation cards. The main Objectives of the activity: • Students will develop skills in measuring, cutting and precision • Students will understand the relation between fractals and recursive methods. • Students will estimate the ratio and portion while cutting • Students will understand complex patterns and creativity in math • Material used - A4 size paper, A4 size coloured papers, Scissor, Glue, Scale
Assessment	
Formative	• Students will show their understanding of the topics taught by completing daily homework assignments. • Participation in class discussions • On-the-spot performance

Interim	For PBL - Group based presentation wherein they will talk about the plans they have analysed and designed. It will give them an opportunity to showcase their communication skills, technological skills and the understanding of the content.
Summative	• Investigation on logarithms • Investigation on Complex Numbers • Chapter/Unit test (Keys and solutions to tests) *(pg no: 10)* • Mid-term and term end exam
Differentiation	
Affirm identity—build self-esteem	• During the transition from grade 10 to 11, we create an environment wherein the learner feels welcomed. As a teacher, I design lesson plans in such a way that it takes care of the pre-IB requirement. • Peer to peer teaching works well with our students. • Encourage students to watch and discuss a lecture/content from Khan academy videos. • Reinforcement on one to one basis creates a good foundation for remedial/revision classes. • The students use Notes and worksheets designed by Christos Nikolaidis for independent learning.

TOK

- Why the symbol of infinity ' ∞ 'the way it exist?
- How is mathematical intuition used as a basis for formal proof? (Gauss' method for adding up integers from 1 to 100.)
- The nature of mathematics. Has "i" been invented or was it discovered?
- Was the complex plane already there before it was used to represent complex numbers geometrically?
- Do the words imaginary and complex make the concepts more difficult than if they had different names?
- The nature of mathematics. Has "i" been invented or was it discovered?
- Why does "i" appear in so many fundamental laws of physics?
- Do proofs provide us with completely certain knowledge?
- What are the different meanings of induction in mathematics and science?

For induction topic, the TOK session is designed, keeping in mind WOKs. The teacher will divide the students into seven groups, each containing two. The students will think about examples from real life situations and share their understanding with the class. TOK session on Complex Number will focus on the discussion " Has 'i ' been invented or was discovered". The student will be asked to make a presentation about his understanding about "why does 'i ' appear in so many fundamental laws of physics?". Discussion on topics like infinity is often an outcome of digression while teaching a conceptual content like geometric progression or L'Hospitals rule for finding limits.

International Mindedness *(questions from IB mathematics guide)*
• What was the contribution of Aryabhatta and al-Khawarizmi in Algebra? Did one mathematicians work influence the other? • The use of several alphabets in mathematical notation (eg first term and common difference of an arithmetic sequence).
The students will thoroughly research the above questions in different groups. They will present their understanding and learning in a group through a power point presentation. Initially, within the group's students will summarize their ideas and learnings.
Resources
• Mathematics Higher Level, Oxford • IBID Press textbook • Kognity • Thinkib • Haese and Haaris • Online notes/lectures(Khan Academy) • Notes by Christos Nikolaidis

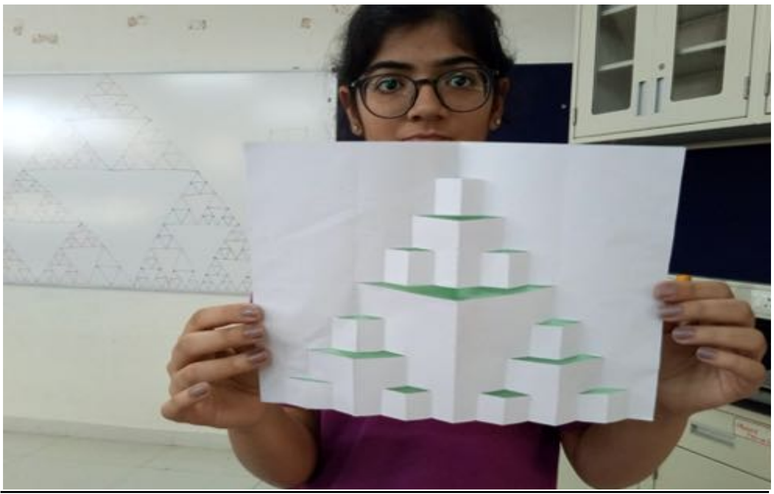
Reflection-considering the planning, process and impact of the inquiry

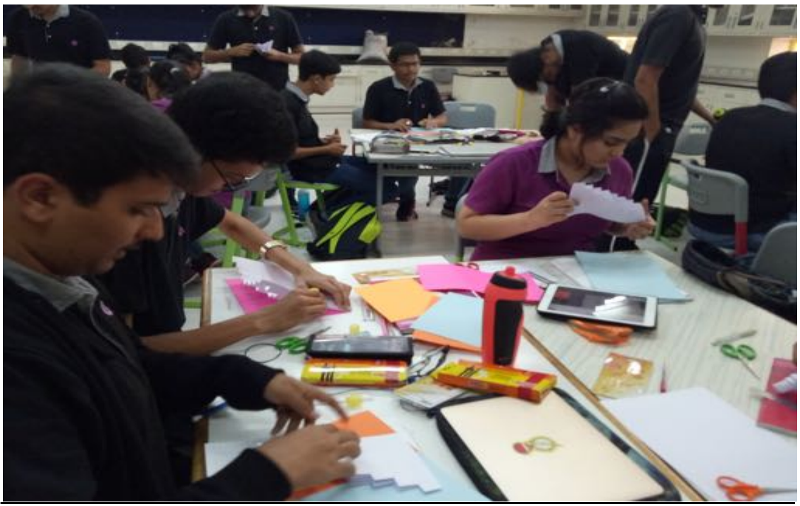
What worked well *List the portions of the unit (content, assessment, planning) that were successful*	**What didn't work well** *List the portions of the unit (content, assessment, planning) that were not as successful as hoped*	**Notes/changes/suggestions:** *List any notes, suggestions, or considerations for the future teaching of this unit*
• **Superannuation Activity** - Design a project, which took care of core IB curriculum (Service and Creativity). Collaborated with language teachers, Financial advisor and Economic teacher. Though the interaction with students was for a short duration, the economics teacher gave a good explanation of superannuation from a global perspective. Analysing the pension plans. • **Logarithms Investigation** - The students understood the formation of logarithmic tables through an investigative approach. This strategy in particularly helped students to learn and unlearn the concepts of logarithms. The worksheet designed for investigation included the assessment of the concepts taught (Logarithmic properties) and simple algebraic skill. • **Fractal Activity** – Ideation of the design of pop-out cards was successful. It was an integration of creativity, conceptual understanding of recursive patterns, non-discrete dimension (unplanned), and geometric progression.	• **Logarithms Investigation**: Due to time constraint, the students could not focus dedicated time to application of logarithms in measuring pH, Earthquake, and more. • **Fractal Activity**- Students could not generate a recursive formula for the activity. • **Superannuation** - Students' inefficacy to work within a group. Not able to complete the task of designing an end product. Time Management. Relying on websites for complete information. Few students did not find creating the end product relevant. • **Permutation and Combination** – Activity designed for P and C did not go well with the student because of the very nature of the activity.	• **Logarithms Investigation** – Few students could not complete the activity in the stipulated time. • **Fractal Activity** – Connect the concept to non-discrete dimensions. • **Superannuation –** Involve different investment plans like mutual funds to make it more interesting for students.

Students Reflection

Activity	Reflection
Fractals-Pop-out cards	Sahil Khajuria *Before the activity, I always thought that art isn't my cup of tea. One more stereotype that I had was that there is no real connection between maths and art. However, after the activity, both of these misconceptions were cleared. I thoroughly understood the concept of fractals, and that too without reading much on the same. But most importantly, I personally loved doing the activity, as it managed to keep me deeply engrossed in it.*
	Vaibhav Ramani *Even after hearing there was going to be some activity in the class, I entered the class thinking that this was going to be a long 1 and a half hours. However, as the class progressed, the way things (about fractals) unfolded themselves seemed challengingly compelling, and kept me anticipating actively what pattern a new fractal will now follow. The activity, though was basically following instructions, ended up having me know what to do to further continue the fractal. This is how the class from start to end, was immersive. Even after, I did think for a while how I was going to use the product I created to generalize geometric patterns. Overall, the class was a good example of how syllabus-covering classes can still be fun.*
	Devashish *The activity was extremely inquisitive with a connection of fractals to kirigami. Personally, I am a beginner in art, yet, I could grasp most of what we learned about fractals. This activity showed me one of many applications of mathematics in real life. In my opinion, the activity was fairly slow - paced at some places, but overall it was never boring and the overall product was something I never thought we could make so simply.*

<u>Fractal Activiy- Popout cards</u>

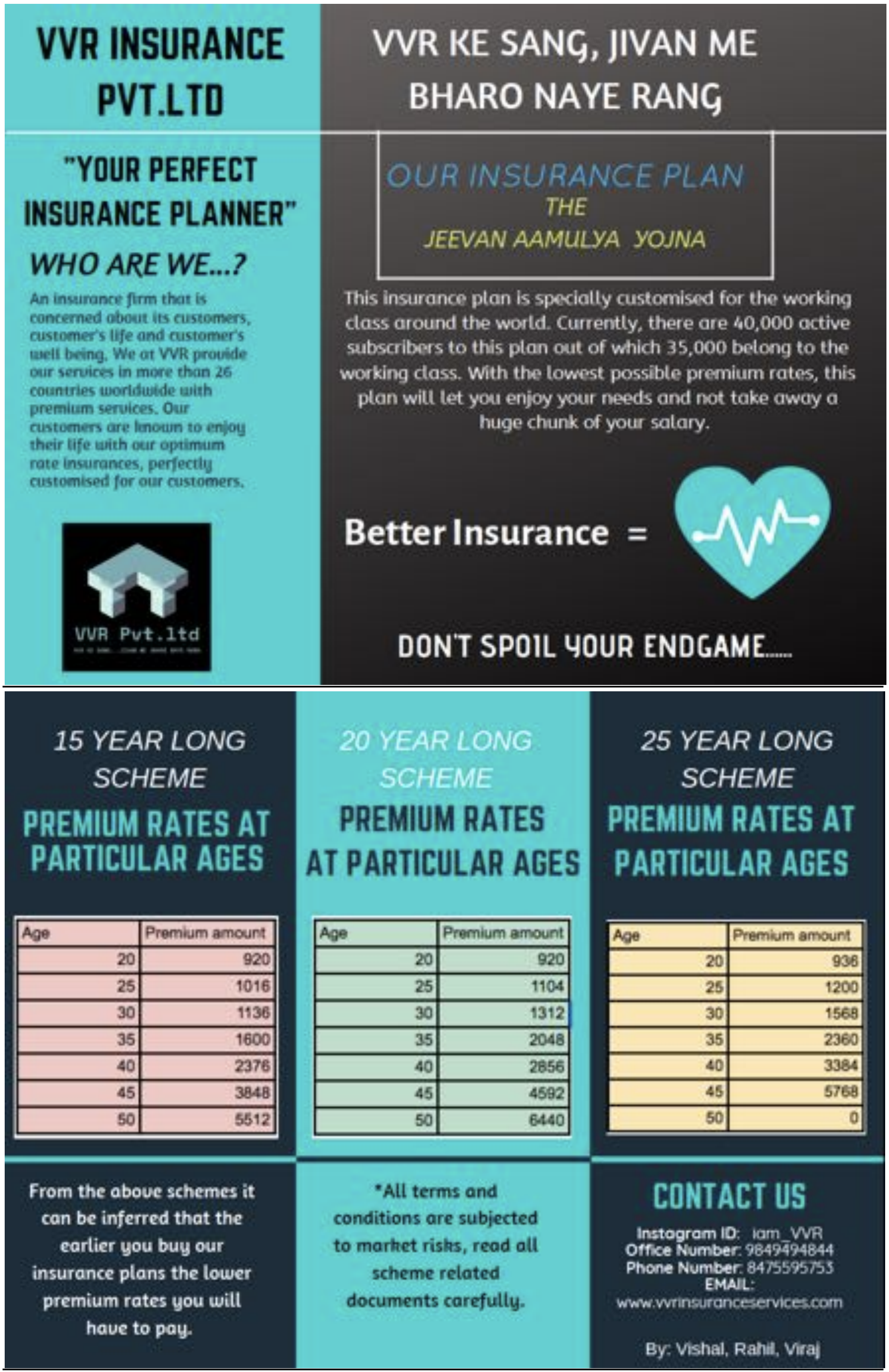




The following links are video clip about students understanding the concept of fractals before they began with the activity. Students are drawing the Sierpinski triangle. Sierpinski triangle is a fractal. It is a self-similar structure that occurs at different levels of iterations.
https://drive.google.com/drive/folders/1JbgmwY4pD8on-VUVI4nnqgHzkAJTOtir

https://drive.google.com/drive/folders/1JbgmwY4pD8on-VUVI4nnqgHzkAJTOtir

Superannuation - Students work



Links to more student work

https://drive.google.com/drive/folders/1BJA4vgxaEYbWbcg6GMM7fYD3hIN2SnEJ

	Topic – Superannuation- Feedback Form			Facilitator - Poorvi Doshi and Kapil Savjani	
		Rarely	**Sometimes**	**Most of the times**	**Always**
	How was the topic introduced? Teacher provides activities/investigation that makes subject matter meaningful.	Activity	Investigation	Chalk and Talk – probing questions	Combination of all three ✓
		1	**2**	**3**	**4**
1	Teacher is prepared for class.				✓
2	Teacher is organized and neat (board work).				✓
3	Teacher plans class time and assignments that help students to solve problem collaboratively and think critically.				✓
4	Teacher is flexible in accommodating for individual student needs.				✓
5	Teacher is clear in giving directions on explaining what is expected on assignments and tests/board exams.				✓
6	Teacher manages the time well.				✓
7	I have learnt about this topic to apply in real life.				✓
8	Teacher is creative in developing activities and lessons				✓
9	Teacher encourages students to speak up and be active in the class.			✓	
10	Teacher makes connections with real life?				✓
11	Teacher make connections with other subjects?				✓

REMARKS

Perseverance required to understand superannuation.

Financial Literacy, especially regarding investment options and annuities is a considerable although small issue.

How do you decide based on information how much premium to change?

Investigation- Binomial theorem
(source:*http://occ.ibo.org/ibis/occ/home/userResourcesNew.cfm?subject=mathl)*

Mathematics HL
Mathematical Investigation on Binomial Theorem

Note : **Duration : 45 min.**

1. In the following exercise, follow the steps as instructed and answer the questions.
2. This is a set of sequential steps, so follow the steps in the sequence. If you skip a step, you may not be able to take the following steps to answer subsequent question.
3. Complete the paper and hand it over to the invigilator with the question paper.

1. Write the expansion of $(a+b)^2$, $(a+b)^3$, $(a+b)^4$, $(a+b)^5$, $(a+b)^6$.

2. Arrange, in each of the above expansion, the terms in such a way that the powers of a are in descending order.

Q. 0 : How many terms are there in the expansion of $(a+b)^2$?

Q. 1 : How many terms are there in the expansion of $(a+b)^3$?

Q. 2 : How many terms are there in the expansion of $(a+b)^4$?

Q. 3 : How many terms are there in the expansion of $(a+b)^5$?

Q. 4 : How many terms are there in the expansion of $(a+b)^6$?

Q. 5 : What is the relation between the power of $(a+b)$ and the number of terms in the expansion ? Do you see any pattern there ?

Q. 6 : From the above mentioned observation, can you predict the number of terms in the complete expansion of $(a+b)^n$? where n is a positive integer.

3. Arrange the numerical coefficients of the terms in expansions of $(a+b)$, $(a+b)^2$, $(a+b)^3$, $(a+b)^4$, $(a+b)^5$, $(a+b)^6$ in the following triangular form :

$n=1$ coeff. 1st term coeff. 2nd term
$n=2$ coeff. 1st term coeff. 2nd term coeff 3rd term
$n=3$ coeff. 1st term coeff. 2nd term coeff 3rd term coeff 4th term
$n=4$ coeff. 1st term coeff. 2nd term coeff 3rd term coeff 4th term coeff 5th term
$n=5$ coeff. 1st term coeff. 2nd term coeff 3rd term coeff 4th term coeff 5th term coeff 6th term
Etc…

Q. 7 Do you see any relation between the numbers in a given row and its previous row ? Express it in words.

Q. 8 Do you see any relation between the numbers in a given row and the numbers in the next row ?

Q. 9 Given the numbers in a row (say n = 5), predict the numbers in the next row n = 6, n = 7, n=8, n=9 and n = 10.

Q. 10 For a given n, what is the sum of the powers of a and b in the expansion ? What pattern do you see there ?

Q. 11 Do you see the same pattern in the all the expansions ? State it in words.

Q. 12 Add the coefficients of all the terms in the expansion $(a+b)$ $(a+b)^2$, $(a+b)^3$, $(a+b)^4$, $(a+b)^5$, $(a+b)^6$ and so on. Note the pattern you observe here.

Binomial Investigation - Student work

Mathematical Investigation on Binomial Theorem

Binomial theorem

1. $(a+b)^2$
$= a^2 + 2ab + b^2$

$(a+b)^3$
$= (a+b)a^2 + 2ab + b^2$
$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$
$= a^3 + 2a^2b + 2b^2a + ab^2 + b^2a^2 + b^3$
$= a^3 + 3a^2b + 3b^2a + b^3$

$(a+b)^4$
$= (a^3 + 3a^2b + 3b^2a + b^3)(a+b)$
$= a^4 + 3a^3b + 3b^2a^2 + ab^3 + ba^3 + 3a^2b^2 + 3b^3a + b^4$
$= a^4 + 4a^3b + 6a^2b^2 + 4b^3a + b^4$

$(a+b)^5$
$= (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)(a+b)$
$= a^5 + 4a^4b + 6a^3b^2 + 4a^2b^3 + b^4 + ab^4 + 4a^3b^2 + 6a^2b^2 + 4ab^4 + b^5$
$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$(a+b)^6$
$= (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)(a+b)$
$= a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5 + a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4$
$+ 5ab^5 + b^6$
$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

2. 3 (2) Q4. 7 (6)
Q1. 4 (3) Q5. Power + 1
Q2. 5 (4) Q6. (n+1)
Q3. 6 (5)

3. n=1   1 1
n=2   1 2 1
n=3   1 3 3 1
n=4   1 4 6 4 1
n=5   1 5 10 10 5 1

Q7 It is the sum of the 2 numbers above it

Q8. Add the two numbers to get the number under it

Q9 n=6   1 6 15 20 15 6 1
n=7   1 7 21 35 35 21 7 1
n=8   1 8 30 56 70 56 30 8 1
n=9   1 9 38 86 126 126 86 38 9 1
n=10   1 10 47 124 212 252 212 124 47 10 1

Q10 n = the sum of a and b

Q11 yes

Q12 2   Pattern is 2^n.
4
8
16
32
64
128

LOGARITHM and EXPONENTIAL INVESTIGATION - Assessment

EXPONENTIAL INVESTIGATION

Consider how an investment can earn continuously compounded interest:

If a principal amount $P is invested at an annual percentage rate r, compounded once a year, the amount in the balance, $A, after one year is given by ____________

We can then have more frequent (quarterly, monthly, daily) compounding interest.

For example, if we have quarterly compounding interest then each quarter will have an effective rate of ____________, which will be compounded 4 times. This means that by the end of the year, the balance will be given by ______________.

If we next consider the situation where there are n compounding's per year, so that the rate per compounding becomes, we then have that the amount in the balance after a year (i.e., after n compounding's) is given by

____

If we allow the number of compounding's n, to increase without bound, we obtain what is known as continuous compounding. We can set up a table of values for the case when n = 1.

n	$\left(1+\frac{1}{n}\right)^{n}$
1	$\left(1+\frac{1}{\_}\right)^{\_} =$
10	$\left(1+\frac{1}{\_}\right)^{\_} =$
100	$\left(1+\frac{1}{\_}\right)^{\_} =$
1000	$\left(1+\frac{1}{\_}\right)^{\_} =$
10000	$\left(1+\frac{1}{\_}\right)^{\_} =$

LOGARITHM INVESTIGATION

Activity Part 1

Approximating the values of basic logarithms of numbers between 1 and 10 without using GDC and log tables.

I) Figure out the value for logarithm of 1 (to the base 10):

II) Approximating the value for logarithm of 2 (to the base 10):
Fill in the following table:

x	2^x		x	2^x
1			6	
2			7	
3			8	
4			9	
5			10	

Using an approximated value of 2^x for x = 10, approximate the value of **log (2):**
(Hint: Use the power rule of logarithms)

III) Approximating the value for logarithm of 3 (to the base 10):
Given that $3^4 = 81$, we can say it

$$3^4 \approx 80 = 8 \times 10$$

$$\_\ \log_{10}(3) = \log_{10}(\_) + \log_{10}(\_)$$

Using appropriate properties, approximate the value of log (3)

$$\log_{10}(3) \approx \_\_$$

IV) Approximate the value for logarithm of 5 (to the base 10):
(Hint: 5 = 10 ÷ 2)

V) Approximate the value for logarithm of 7 (to the base 10):

$$7^4 = 2041$$

$$7^4 = 3 \times \_ \times \_$$

$$\_\ \log_{10}(7) = \log_{10}(\_) + \log_{10}(\_) + \log_{10}(\_)$$

$$\log(7) \approx \_\_\_$$

VI) Fill in the following table with approximated values:

$\log_{10}(1)$			$\log_{10}(6)$	
$\log_{10}(2)$			$\log_{10}(7)$	
$\log_{10}(3)$			$\log_{10}(8)$	
$\log_{10}(4)$			$\log_{10}(9)$	
$\log_{10}(5)$			$\log_{10}(10)$	

Using these logarithms, we can calculate values of other logs such as $\log_5 8$. But HOW? You might be wondering. For this we would construct a very interesting scale method in the following part.

Activity Part 2) Creation of Logarithmic Scale

- To create this scale, we would be using the table created in the above section. Draw a 10 cm line below. Divide the scale according to the values of logarithms given above and mark with appropriate point. For example, Log (1) = 0, hence at zero cm on the scale mark the point 1, and log (2) = 0.3 (approximately) hence at around 3 cm (as our scale is of 10 cm) on the scale mark the point 2.
- Draw a linear scale of 10 cm below and write your reflections about the nature of logarithmic scale compared to linear scale.
- Now to calculate $\log_5 8$, you would draw both the scales side by side and look for the corresponding value of log 5 and log 8. Use these two corresponding values and the change of base property to calculate $\log_5 8$.
- Do try out calculating other logarithms of your choice with the method given above.

Activity Part 3) Reflection

Reflect on the following questions:

1) How close is the accuracy of logarithmic values we calculated in the first part of the activity?
2) Is there a method to get closer approximation?
3) Reflect on the accuracy of the logarithms calculated in the second part.
4) How could you improve you accuracy of calculating logarithms in part 2.
5) Find out the applications of logarithms in measuring pH, Earthquake and more.

Logarithm Investigation- Student work

Devanshee

EXPONENTIAL INVESTIGATION

Consider how an investment can earn continuously compounded interest:

If a principal amount $P is invested at an annual percentage rate r, compounded once a year, the amount in the balance, $A, after one year is given by

$\$A = P(1+r)^1$

We can then have more frequent (quarterly, monthly, daily) compounding interest.

For example, if we have quarterly compounding interest then each quarter will have an effective rate of $\frac{r}{4}$, which will be compounded 4 times. This means that by the end of the year, the balance will be given by $P(1+\frac{r}{4})^4$.

If we next consider the situation where there are n compounding's per year, so that the rate per compounding becomes, we then have that the amount in the balance after a year (i.e., after n compoundings) is given by $P(1+\frac{r}{n})^n$

If we allow the number of compounding's n, to increase without bound, we obtain what is known as continuous compounding. We can set up a table of values for the case when r = 1.

n	$\left(1+\frac{1}{n}\right)^n$
1	$\left(1+\frac{1}{1}\right)^{1} = 2$
10	$\left(1+\frac{1}{10}\right)^{10} = 2.59374$
100	$\left(1+\frac{1}{100}\right)^{100} = 2.70481$
1000	$\left(1+\frac{1}{1000}\right)^{1000} = 2.71692$
10 000	$\left(1+\frac{1}{10\,000}\right)^{10\,000} = 2.71815$

↓ 'e'

$\left(1+\frac{1}{n}\right)^n = 2.718$ 'e' ; natural log

VI) Fill in the following table with approximated values:

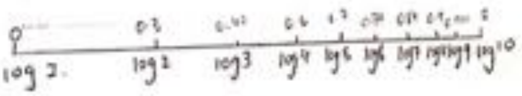
log(1)	0	log(6)	0.77
log(2)	0.3	log(7)	0.8439
log(3)	0.4775	log(8)	0.9
log(4)	0.6	log(9)	0.94
log(5)	0.7	log(10)	1

$\frac{6}{10} = 1.6$

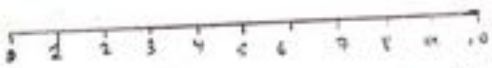
Using these logarithms we can calculate values of other logs such as $\log_5 8$. But HOW? You might be wondering. For this we would construct a very interesting scale method in the following part.

Activity Part 2) Creation of Logarithmic Scale

To create this scale we would be using the table created in the above section. Draw a 10 cm line below. Divide the scale according to the values of logarithms given above and mark with appropriate point. For example, Log (1) = 0, hence at zero cm on the scale mark the point 1, and log (2) = 0.3 (approximately) hence at around 3 cm (as our scale is of 10 cm) on the scale mark the point 2.



Draw a linear scale of 10 cm below and write your reflections about the nature of logarithmic scale compared to linear scale.



logarithmic scale first has a larger gap between two logs while linear scale has equal gaps. Logarithmic scale has many values at the end of the scale.

Now to calculate $\log_5 8$, you would draw both the scales side by side and look for the corresponding value of log 5 and log 8. Use these two corresponding values and the change of base property to calculate $\log_5 8$.

$$\frac{\log 8}{\log 5} = \frac{0.9}{0.7} = 1.28571$$

LOGARITHM ACTIVITY

Activity Part 1

Approximating the values of basic logarithms of numbers between 1 and 10 without using GDC and log tables.

I) Figure out the value for logarithm of 1 (to the base 10):

$\log_{10} 1 = 0$

II) Approximating the value for logarithm of 2 (to the base 10):

Fill in the following table:

x	2^x	x	2^x
1	2	6	64
2	4	7	128
3	8	8	256
4	16	9	512
5	32	10	1024

Using an approximated value of 2^x for x = 10, approximate the value of log (2):
(Hint: Use the power rule of logarithms) →

$2^{10} = 1024$
$2^{10} \simeq 1000$
$\log_{10} 2^{10} = \log_{10} 1000$
$10 \log_{10} 2 = \log_{10} 10^3$
$10 \log_{10} 2 = 3 \log_{10} 10$
$\log 10^2 = \frac{3}{10}$
$\log_{10} 2 \approx 0.3$

III) Approximating the value for logarithm of 3 (to the base 10):
Given that $3^4 = 81$, we can say it

$\log_3 81 = 4$

$3^4 \approx 80 = 8 \times 10$

$4 \log(3) = \log(8) + \log(10)$

Using appropriate properties, approximate the value of log (3)

$\log_{10} 3^4 \approx \log_{10} 80$ $\quad \log_{10} 2^3 + \log$
$4 \log_{10} 3 \approx 3 \log_{10} 2 + 1$
$\log(3) = 0.475$
$4 \log_{10} 3 = 1.9$
$= \frac{1.9}{4} = 0.475$

IV) Approximate the value for logarithm of 5 (to the base 10):
(Hint: 5 = 10 ÷ 2)

$\log_{10} 5 = 9765625$
$\log_{10} 5 \simeq 1000000$
$\log_{10} 5^{10} \approx \log_{10} 10^6$
$10 \log_{10} 5 \approx 6 \log_{10} 10$
$\log_{10} 5 \approx 0.6$

V) Approximate the value for logarithm of 7 (to the base 10):

$7^4 = 2401$
$7^4 \approx 3 \times 8 \times 10$
$4 \log(7) = \log(3) + \log(8) + \log(10)$

log(7) =

$\log_{10} 7^4 \approx \log_{10} 3 + \log_{10} 2^3 + \log_{10} 10^2$
$4 \log 7 \approx \log_{10} 3 + 3 \log_{10} 2$
$3(0.3) + 1 = 2.9 + 0.475 = \frac{3.375}{4}$
≈ 0.84375

Do try out calculating other logarithms of your choice with the method given above.

Activity Part 3) Reflection

Reflect on the following questions:

1) How close is the accuracy of logarithmic values we calculated in the first part of the activity?
2) Is there a method to get closer approximation?
3) Reflect on the accuracy of the logarithms calculated in the second part.
4) How could you improve you accuracy of calculating logarithms in part 2.
5) Find out the applications of logarithms in measuring pH, Earthquake and more

1) The logarithmic value is very close in the first part of the activity because there is 0.01 or so to the second decimal there is little change.

2) I do not know other method to get closer approximation because the values in this is very close to actual.

3) Accuracy of logarithms in second part is not accurate because we get values like $\log_{10} 2$ is 0.477121 which is very difficult to identify on the scale.

4) To improve accuracy in logarithms of part 2. we could have the scale with more divisions so that we can place the value of the log accurately

5) logs can be used in pH as log negative base 10 of an element ion activity can determine the pH. ion activity can be the concentration.
To find earthquake on Richter scale we do ~~log~~ $\log\left(\frac{\text{measure of amplitude to the earthquake wave}}{\text{smallest detectable wave}}\right)$

(can be used in interest rates, GDP of a year and the next.)

Student work - International Mindedness

https://drive.google.com/drive/folders/1pgDuNUn-akYvu81BDWsr5JkeTMP-I0Uo

Lesson plans

Sequence and Series

Logarithms

Binomial Theorem

Permutation and Combination

Mathematical Induction

Complex Numbers

Assessment

THE GALAXY SCHOOL
Mathematics: IBDP HL
Assessment 2 : Week 1 March 2019 [Term 1/year 1]
TOPIC: Sequences and Series, Exponents and Logarithms
Max. Marks: 37

Duration: 42 mins

1. [Maximum mark: 7]

In an arithmetic sequence, the fifth term is 27, the sixteenth term is 115, while the **n**-th term is 155.

Find

(a) The value of **n** [4 marks]

(b) The sum of the first $n-1$ terms. [3 marks]

2. [Maximum mark: 7]

Let $S = 1 + \frac{3}{k} + \frac{9}{k^2} + \frac{27}{k^3} + \cdots$. Calculate

(a) the value of k given that $S = 4$ [3 marks]

(b) the possible values of k given that the infinite series S diverges. [3 marks]

3. [Maximum mark: 4]

Solve for x: $4^x - 9 \times 2^x + 8 = 0$

4. [Maximum mark: 6]

The third term of a geometric sequence is -108 and the sixth term is 32. Find

(a) The common ratio [3 marks]

(b) The first term [1 mark]

(c) The sum to infinity [2 marks]

5. [Maximum Marks 6]

Solve for x:

$$2\log_3(x-3) = 2 - \log_{1/3}(x+1)$$

6. [Maximum mark: 7]

An arithmetic sequence has first term **a** and a common difference **d**. The 3rd, 4th and 7th terms of the arithmetic sequence are the first three terms of a geometric sequence.

(a) Show that $a = -\frac{3}{2}d$. [4 marks]

(b) Find the common ratio of the geometric sequence. [3 marks]